Optimization Based Synthesis Methods to Design Multi-Channel Sampled Fiber Bragg Gratings with Phase-Shift

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Abstract

In this paper, two synthesis methods are proposed for designing multi-channel sampled gratings with phase-shifts. The first synthesis method uses non-equally separated π-phase shifts and the second uses variable but equally separated phase shifts in sampling period of fiber grating. In both methods, the Fourier transform derived from the phase shifts is related with the reflection spectrum of the fiber Bragg grating. It is demonstrated that a reflection spectrum with desired number of channels can be easily synthesised using a general optimization method. Besides, a comparison is given between these two methods according to the target reflection spectrum.

1. Introduction

The fiber Bragg grating technology (FBG) has contributed very important developments to both the telecommunications and fiber optic sensing fields such as tunable semiconductor reflectors [1-2], chromatic dispersion compensators [3-4], multi-channel optical filters [5-9], multi-wavelength fiber lasers [10-11] and temperature measurements [12]. Multi-channel optical filters realized using sampled fiber Bragg gratings have recently attracted great interest in wavelength division multiplexing (WDM) systems of the modern superfast lightwave communication networks. This kind of filters based on utilizing either amplitude-sampled or phase-sampled FBG [5-9], the superimposed FBG [13] and the Talbot-effect based FBG [14-15] have been demonstrated.

The phase-sampled FBGs having the phase shifts located in the sampling period of grating, requires a lower level for the maximum refractive index modulation depth with respect to the other fiber Bragg grating filters since it has higher refraction efficiencies. In addition, a high channel number and good channel uniformity can be produced by optimizing the phase sampling function that defines the phase shifts.

In this paper, we demonstrate that a grating spectrum matched perfectly to the desired number of channels and the uniformity of the channel intensities can be achieved. Two different approaches are proposed to set up the phase sampling function. In the first synthesis method, constant π phase shifts are nonuniformly located in the sampling period of grating. The second synthesis method includes variable phase shifts located uniformly in the sampling period of grating. Note in determining the phase sampling function that the phase shifts remain constant at π and the phase transition positions are variable for the first method; but the amplitudes of the phase shifts are variable and the phase transition positions remain constant for the second method. Fig.1. shows the illustration of the phase sampling for a period of fiber grating with grating period Λ.

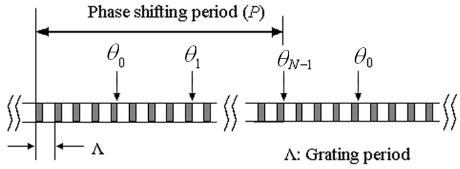

Fig. 1. Illustration of phase-shifted FBG

2. Theory

Effective refractive index variation in an ordinary FBG can be described as:

$$n(z) = n_{co}(z) + \Delta n_{co} \cos\left(\frac{2\pi z}{\Lambda} + \phi(z)\right) \quad (1)$$

Here, n_{co} is the average refractive index of the fiber core, Δn_{co} is the modulation depth of the refractive index, $\phi(z)$ is the grating phase and z is the position along the grating. A uniform FBG has a single resonant peak in its reflection spectrum. The phase shifts, added to a uniform grating, affect its reflection spectrum. A phase sampled FBG is a grating that its profile is modulated along the fiber by a periodic phase sampling function s(z). The refractive index variation described in Eq.(1) can be rearranged with a periodic phase sampling function as:

$$n(z) = n_{co}(z) + \Delta n_{co} \operatorname{Re}\left\{\exp\left[i\left(\frac{2\pi z}{\Lambda} + \phi(z)\right)\right] s(z)\right\} \quad (2)$$

The grating phase $\phi(z)$ may contain a chirp in the grating. The dispersion compensation and dispersion slope compensation can be realized by sampled-chirped FBG. However, only periodic phase sampling functions excluding the effect of chirp are considered in this paper.

In order to set up a periodic phase sampled function in the first synthesis method, it can be considered that a number of π phase shifts nonuniformly located along the fiber are inserted within the sampling period. This type of phase sampling is called Dammann sampling or binary phase sampling. The target of the phase sampled FBG synthesis using Dammann phase sampling is to minimize the number of π phase shifts.

The Dammann phase sampling function of each sampling period having K times π phase shifts is given using the rectangular (rect) function as [16]:

$$s(z) = \sum_{n=0}^{K} \exp(i\theta_n) rect\left[\frac{z - \left(\frac{z_{n+1}}{2}\right)}{z_{n+1} - z_n}\right] \quad (3)$$

Here, $\theta_n = n\pi$ and z_n denotes the phase shifting transition points with $n=0,1,2,..K$. Using the Fourier analysis, the complex Fourier coefficients of $s(z)$ can be expressed as:

$$S_m = -\frac{1}{2im\pi} \sum_{n=0}^{K} (-1)^n \left[\exp(-2i\pi m z_{n+1}) - (\exp(-2i\pi m z_n)\right] \quad \text{for } m \neq 0.$$

$$S_0 = \sum_{n=0}^{K} (-1)^n \left(z_{n+1} - z_n\right) \quad \text{for } m=0. \quad (4)$$

In the second synthesis method, a number of phase shifts located uniformly along the fiber are inserted within the sampling period. But, the amplitudes of the phase shifts are not equal and alternate in the range (0,π). Hence, the sampling function is given by:

$$s(z) = \sum_{n=0}^{K} \exp(i\theta_n) rect\left[\frac{z - \left(\frac{z_{n+1}}{2}\right)}{z_{n+1} - z_n}\right] \quad (5)$$

and its Fourier transform is,

$$S_m = \frac{1}{2im\pi} \sum_{n=0}^{K} \left[\exp(i\theta_n)\left[\exp(-2i\pi m n / K) - \exp(-2i\pi m(n+1)/K)\right]\right] \quad \text{for } m \neq 0.$$

$$S_0 = \frac{1}{K} \sum_{n=0}^{K} \exp(i\theta_n) \quad \text{for } m=0. \quad (6)$$

There is an analogy between the periodic sampling function and spatial optical diffractive element. This analogy is also set up between the reflection spectrum of the FBG and the periodic phase sampling function. The reflection spectrum of the phase sampled FBG can be approximated as the Fourier transform of the grating profile. In order to make the peak reflectivity of the phase sampled FBG are same for all WDM channels, a phase profile that its Fourier transform is similar to the reflection spectrum must be determined. The required phase profile for both synthesis methods can be designed with a nonlinear optimization method using Eq (4) and (6), respectively. The cost function can be defined with Eq (7) and solved using a multidimensional minimization algorithm available in Matlab.

$$T = \sum_{-M}^{M} \left[\left|S_m\right|^2 - R_0\right]^2 \quad (7)$$

Here, R_0 is a series with constant elements and corresponds to a target reflectivity which can be calculated by η/N. η is the target diffraction efficiency and N is the number of channels. The algorithm starts with an initial set of phase profile changes θ_n iteratively to minimize T.

3. Simulation Results

The parameters of the gratings used in the numerical simulations to synthesis a periodically phase-shifted FBG are; an effective core mode index of 1.445, a resonance wavelength of 1.55 µm and a total fiber grating length of 10 mm. In this section, the various syntheses are carried out using firstly the Dammann phase sampling having constant π phase shifts and then periodical phase sampling having variable phase shifts. The results are explained with the help of the graphs.

3.1. Synthesis with Dammann Phase Sampling

Firstly, a synthesis with 9 channels were realized using a Dammann phase sampling function with N=6 phase shifts in each sampling period P. At the beginning of the synthesis, an initial solution z_n containing random phase positions is chosen. After inserting the initial solution in Eq (4), the cost given in Eq (7) is determined. Using a multidimensional minimization algorithm, a new solution which reduces the cost is generated. The minimization algorithm is run until the cost becomes acceptable and thus a final solution is obtained.

The Dammann phase sampling function, its Fourier transform and the reflection spectra of the FBG that are obtained by the synthesis for a phase shifted FBG with 9 WDM channels, are presented in Fig. 2(a), (b) and (c), respectively. The period of the sampling function is normalized to 1. So, all of the phase transition positions exist between 0 and 1. The reflection spectrum is calculated using the transfer matrix method and all the channels which their sampling periods correspond to P=1 mm are located to the spectrum with 100 GHz separations.

It is easily understood from Eq(4) that $\left|S_m\right| = \left|S_{-m}\right|$. The optimization process run for the synthesis including m=0,1,2,…M number of Fourier coefficients generates N=2M+1 number of channels. Therefore, it is seen in Fig. 2(b) that 9 channels can be obtained by the Fourier analysis in the synthesis carried out for M=4.

When the number of channel is desired to be even as N=2M, there should be a transition symmetry in Dammann phase sampling function with respect to the center point of the normalized sampling function. In this case, all of the odd Fourier coefficients of the phase sampling function become zero and the phase sampling period P of FBG doubles. The Dammann phase sampling function, its Fourier transform and the reflection spectrum of the FBG that are obtained in an 8 channels synthesis are shown in Fig. 3(a), (b) and (c), respectively.

New designs including higher number of channels can be made by optimizing the Dammann sampling function. Phase transition data obtained for different designs are given in Table 1.

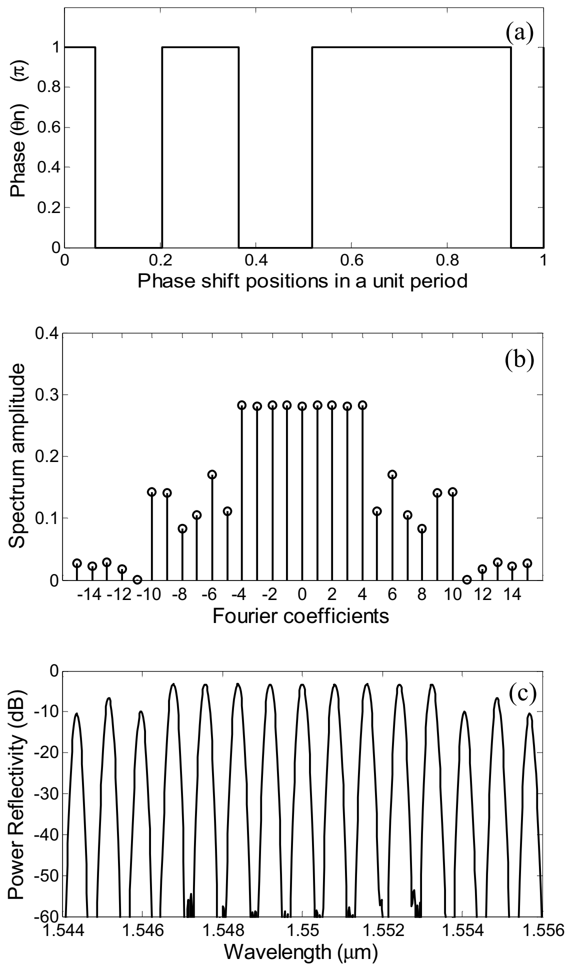

Fig. 2. (a) The Dammann phase sampling function, (b) its Fourier transform and (c) reflection spectrum for 9 channels.

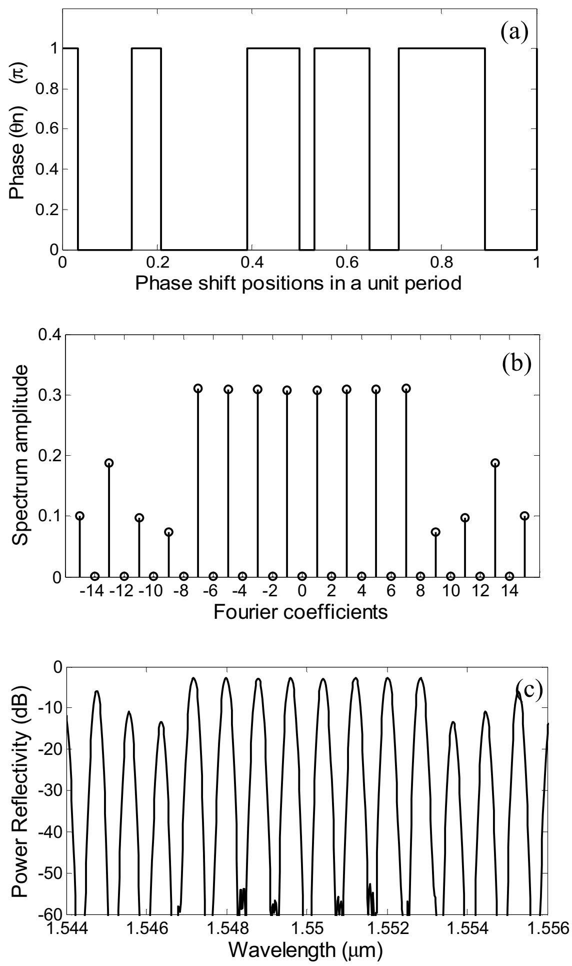

Fig. 3. (a) Dammann phase sampling function, (b) its fourier transform and (c) reflection spectrum for 8 channels.

Fig. 4 shows the reflection spectra of the FBG with 15 and 16 channels, respectively. In both figures, the reflectivities of the all channels are nearly same and are distributed uniformly. In this study, the length of the FBG was kept as 10 mm for the all syntheses. The reflection intensities of the channels can be increased by making the FBG longer.

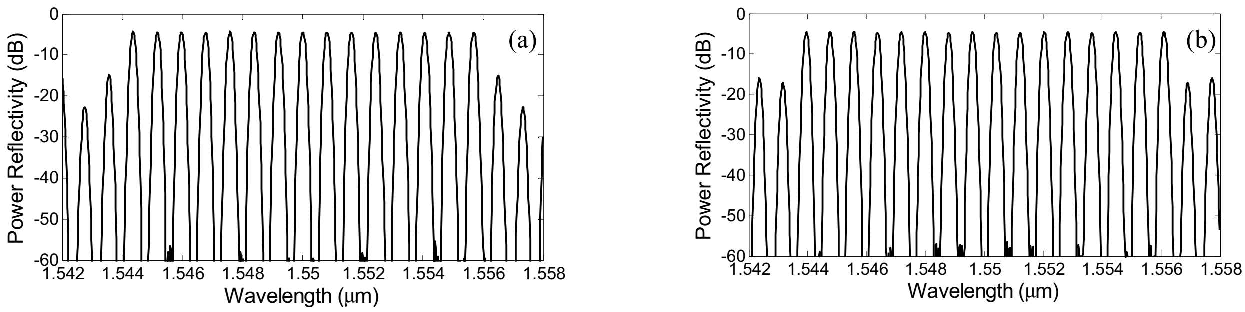

Fig. 4. The reflection spectra of Dammann phase sampled FBG for (a) 15 channels, (b) 16 channels.

3.2. The Synthesis with Variable Phase Sampling

The variable phase sampling function defined in Eq. (6) includes variable phase shifts that are equally separated in each sampling period. In this synthesis method, the Fourier coefficients of the variable phase sampling function given in Eq. (6) are optimized for a target spectrum. When Eq. (6) is analyzed carefully, it is understood that no symmetry relation exists between Fourier coefficients. Therefore, the variable phase function should be symmetric with respect to the center point of the sampling period, as given in Eq. (8):

$$\theta_{n-\frac{K}{2}} = \theta_n \qquad \text{for} \qquad \frac{K}{2} \le n \le K \qquad (8)$$

The variable phase sampling function, its Fourier transform and the reflection spectrum obtained by the synthesis for a FBG with 9 channels are given in Fig. 5(a), (b) and (c), respectively.

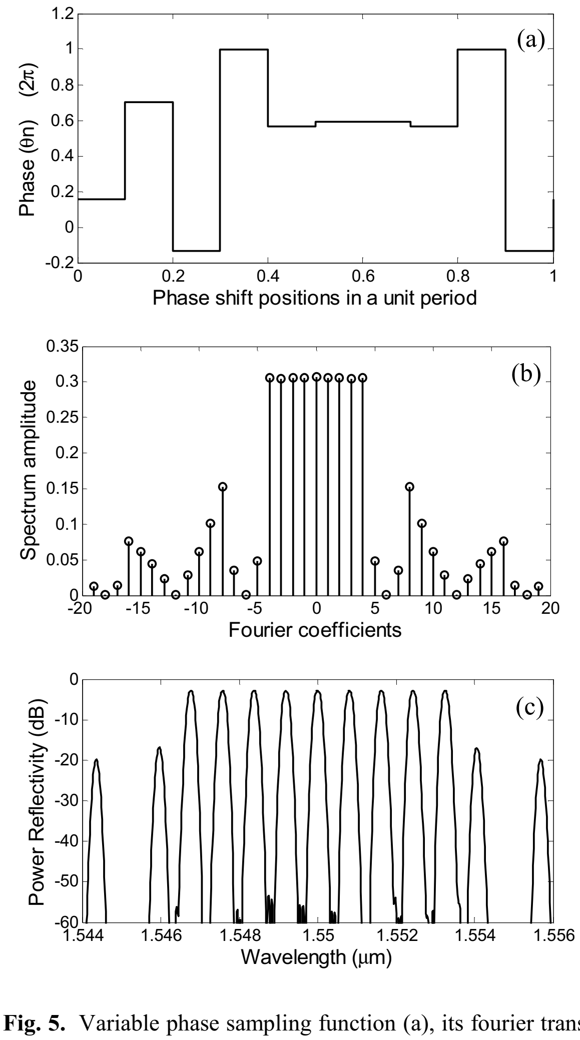

Fig. 5. Variable phase sampling function (a), its fourier transform (b) and reflection spectrum (c) for 9 channels.

When the number of channel is desired to be even, it will be worthless to change the target reflectivity in the optimization process. Instead of this, it is enough to shift the phase of the variable phase sampling function with amount of π or –π, in each sampling period as given Table.1. In Fig. 6, Fourier and the reflection spectra of a FBG with 8 channels are shown as a result of an optimized synthesis.

Table 1. Phase sampling functions for 8 and 9 channels

Dammann Phase Sampling			Variable Phase Sampling		
Transition points	8 channel	9 channel	Phase (2π)	8 channel	9 channel
Z_1	0.0322	0.0658	θ_1	0.8519	0.1568
Z_2	0.1469	0.2045	θ_2	1.0000	0.7012
Z_3	0.2089	0.3647	θ_3	0.2097	-0.1323
Z_4	0.3911	0.5184	θ_4	0.7335	0.9971
Z_5	0.5000	0.9332	θ_5	0.8210	0.5661
Z_6	0.5322	1.0000	θ_6	0.8210	0.5928
Z_7	0.6469		θ_7	0.7335	0.5928
Z_8	0.7089		θ_8	0.2097	0.5661
Z_9	0.8911		θ_9	1.0000	0.9971
Z_{10}	1.0000		θ_{10}	0.8519	-0.1323
Z_{11}			θ_{11}	$\theta_1+\pi$	0.7012
Z_{12}			θ_{12}	$\theta_2+\pi$	0.1568
			θ_{13}	$\theta_3+\pi$	
			θ_{14}	$\theta_4+\pi$	
			…	...	

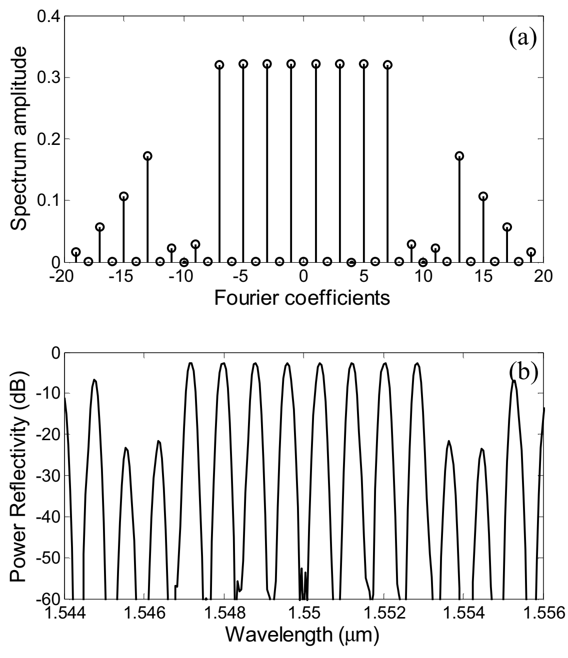

Fig. 6. (a) Fourier transform of the variable phase sampling function and (b) reflection spectrum for 8 channels.

Different synthesis can be carried out to generate more number of channels. Fig.7 shows the reflection spectra of the FBGs with 15 and 16 channels, respectively.

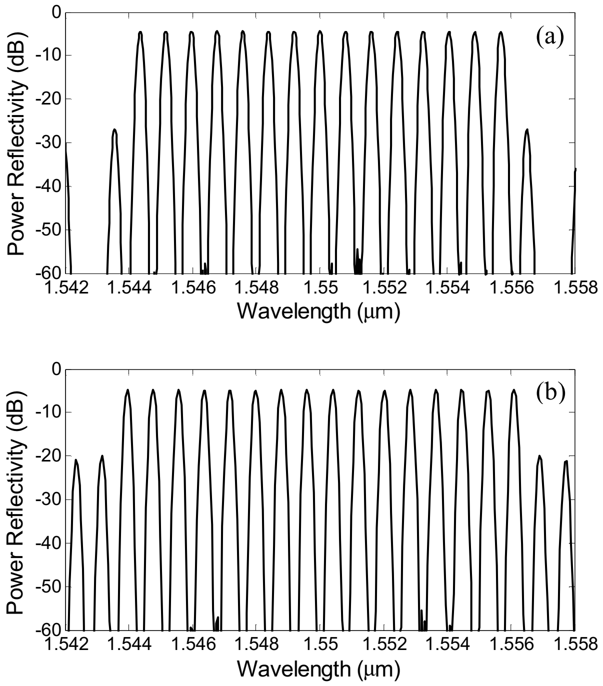

Fig. 7. The reflection spectra of variable phase sampled FBG for (a) 15 channels, (b) 16 channels.

4. Conclusions

In this study, two different methods were proposed to design the phase sampled FBGs and their simulation results were presented. In the first method based on the Dammann phase sampling, several syntheses with an odd and even number of channels have been performed. Using the second method based on the variable phase sampling, more syntheses were realized for the same number of channels. The data resulting from the synthesised phase sampling functions were presented in Table 1 for both phase sampled FBG design methods and for 8 and 9 channels. When this data was compared for the case of 9 channels, it was noticed that 5 points per sampling period for the Dammann phase sampling and 12 points per sampling period for the variable phase sampling are required. When it was compared for 8 channels, it was seen that 10 points in the first method and 20 points in the second method per double sampling period (2xP) are required, respectively. As a result, it was understood that the Dammann phase sampling function can be synthesised using less phase shifts in contrast to the variable phase sampling method.

5. References

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