Comparative Analysis of the Effect of Optical Fiber Spin Profiles on Polarization Mode Dispersion

Şerif Ali Sadık[1], Arif Başgümüş[1], Fırat Ertaç Durak[1], Sait Eser Karlık[2], Ahmet Altuncu[1]

[1] Dumlupınar University, Faculty of Engineering, Department of Electrical and Electronics Eng., Kütahya, Turkey
serifali.sadik@dpu.edu.tr, arif.basgumus@dpu.edu.tr, firat.durak@dpu.edu.tr, altuncu@dpu.edu.tr
[2] Uludağ University, Faculty of Engineering, Department of Electrical and Electronics Eng., Bursa, Turkey
ekarlik@uludag.edu.tr

Abstract

In long-haul and high-bit-rate optical fiber transmission systems Polarization Mode Dispersion (PMD) is one of the main reasons for pulse broadening. In order to reduce PMD-based problems, current studies rely on novel PMD compensators and low-PMD fiber productions. In this paper, after describing theoretical information about PMD and optical fiber spinning process, different spin profiles are examined. Comparative analysis of spin profiles is made and spin parameters giving successful results for low-PMD fibers are obtained for each spin profile. Simulation results show that for fixed spin period of *1 m 2.76 turns/m* and *5.64 turns/m* are proper spin amplitude values for sinusoidal and amplitude modulated (AM) spin profiles, respectively, while *3.40 turns/m* and *3.75 turns/m* are recommended spin amplitudes for triangular and trapezoidal spin profiles, respectively. Using these parameters, related spin profiles give better differential group delay (DGD) reduction results individually.

1. Introduction

Light in circularly symmetrical single mode optical fibers travels as two orthogonally polarized modes. Due to perturbations in circular symmetricity of the fiber, these two modes propagate with different phase velocities and a time delay occurs between them. This delay is called as differential group delay (DGD). DGD leads to pulse broadening at the receiver end. This phenomenon is called as PMD and studied as a performance limiting issue [1].

In order to reduce PMD-based issues, researchers are focusing on feasibility and realization of PMD compensators and low-PMD fibers. The second case deals with introducing controlled polarization mode coupling by fiber spinning. Fiber spinning process has been used in fiber manufacturing and has proven to be a good technique to reduce PMD [2]–[15].

Aim of this study is to present the theory of PMD in spun fibers and comparative analysis of different spin profiles. Second section reviews the coupled mode theory of polarization modes required for understanding birefringence, DGD and PMD phenomena. In the third section we introduce fiber spinning process and different fiber spinning profiles as constant and periodic functions, e.g. sinusoidal, triangular, trapezoidal and amplitude modulated – suppressed carrier (AM-SC) spinning. In the last section, analytical solutions of PMD in spun fibers are obtained and results are discussed comparatively for different spin profiles.

2. Coupled-Mode Equations

Light travels in an optical fiber as two orthogonally polarized modes. In ideal fibers, these two modes propagate with the same phase velocity. However, in real fibers there are some perturbations affecting the circular symmetry of the fiber. Due to these perturbations and some external effects, e.g. stress, twist, and bending, the two orthogonal modes propagate at different speeds because of the difference between their refractive indices. The difference between propagation constants of two modes is called the birefringence and is given by

$$\Delta\beta = \beta_s - \beta_f = \frac{\omega(n_s - n_f)}{c} \tag{1}$$

where β_s and β_f are the propagation constants of slow and fast axis, respectively, n_s and n_f are refractive indices of slow and fast axis and c is the speed of light and ω is the angular frequency of light [16]. Due to birefringence, DGD occurs between slow and fast modes. The PMD of a fiber at position z in the fiber is described as the DGD between two modes in a unit length

$$\tau_\omega = \frac{d(\beta_s - \beta_f)}{d\omega} \tag{2}$$

For constant birefringence, the total DGD after travelling in a fiber of length L is given by

$$\tau = \tau_\omega L \tag{3}$$

(3) is valid for constant birefringence, i.e. for the short fiber regime. However for longer fibers with random birefringence DGD scales with the square root of the fiber length [8].

Without any perturbation due to birefringence, the electric field of a fiber has the form

$$\mathbf{E}_n(x,y,z) = \mathbf{e}_n(x,y)e^{-i\beta z}, \quad n = 1,2 \tag{4}$$

where $\boldsymbol{e}_n(x,y)$ is the electric field distribution, n=1, 2 shows two polarized modes. Without perturbation two modes propagate with same β propagation constant.

Electric field with perturbation can be formed as a linear combination of electric fields of two orthogonally polarized modes as

$$\mathbf{E}(x,y,z)=\sum_n A_n(z)\mathbf{e}(x,y)e^{-i\beta z} \tag{5}$$

where $A_n(z)$ are complex coefficients representing amplitudes of two modes [9].

Complex electric field amplitudes $A_1(z)$ and $A_2(z)$ of the spun fiber can be described by the coupled mode equation

$$\frac{d}{dz}\mathbf{A}=i\kappa\mathbf{A} \tag{6}$$

where κ is a 2x2 coupling coefficient matrix. This matrix is given by

$$\kappa=\begin{pmatrix} 0 & \frac{1}{2}\Delta\beta e^{2i\int_0^z \alpha(z')dz'} \\ \frac{1}{2}\Delta\beta e^{-2i\int_0^z \alpha(z')dz'} & 0 \end{pmatrix} \tag{7}$$

where $\Delta\beta$ is the birefringence and $\alpha(z)$ is the spin profile function. One can get the Jones matrix with integrating (6) with initial conditions of $A_1(0)=1$ and $A_2(0)=0$ as

$$J(z)=\begin{pmatrix} A_1(z) & -A_2^*(z) \\ A_2(z) & A_1^*(z) \end{pmatrix} \tag{8}$$

PMD of a spun fiber can be obtained from Jones matrix as [8], [9]

$$\tau(z)=\frac{2}{L}\sqrt{\left|\frac{dA_1(z)}{d\omega}\right|^2+\left|\frac{dA_2(z)}{d\omega}\right|^2} \tag{9}$$

With initial conditions of $A_1(0)=1$ and $A_2(0)=0$, solutions of $A_1(z)$ and $A_2(z)$ can be computed as

$$\begin{aligned} A_1(z)&=1 \\ A_2(z)&=\frac{\Delta\beta}{2}\int_0^z e^{-2i\Theta(z')dz'} \end{aligned} \tag{10}$$

where $\Theta(z)=\int_0^z \alpha(z')dz'$. One can get the DGD of a spun fiber using (9) as

$$\tau(z)=\tau_\omega\left|\int_0^z e^{-2i\Theta(z')dz'}\right| \tag{11}$$

3. The Spinning of the Transmission Fiber

We consider a fiber with a constant birefringence and assume that this fiber is divided into short segments. The fast axis of one segment is aligned with the slow axis of the following segment. Thus, accumulated DGD of one segment is compensated with the DGD of the second segment as shown in Fig. 1. This mechanism may provide a good DGD reduction in theory but this assumption needs rapid rotations and that kind of rotations are not applicable in practice. However researches on spun fibers show that even a slowly rotated periodic spin profile leads to a DGD reduction [9], [17].

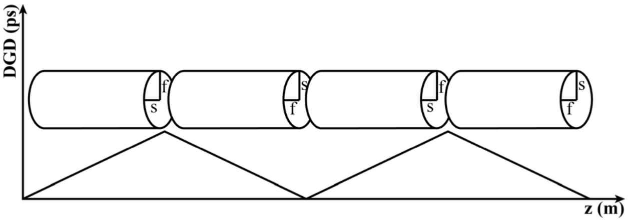

Fig. 1. DGD with periodic fiber spin

There are two main types of fiber spinning in the literature. First one is the unidirectional spin being applied in one direction with a constant spin rate [2], [17]. The second spin type is the bidirectional spin, where the fiber is spun back and forth alternately, based on a periodic spin function. The periodic spin functions are suggested as sinusoidal, triangular, trapezoidal or modulated function in the literature [8], [17].

3.1. Unidirectional Spinning

The unidirectional, i.e. constant, spinning is simpler and easier than the bidirectional one. PMD reduction of the constant spinning is inversely proportional with the beat length. The beat length L_B is the period over which the state of polarization (SOP) of light changes along the fiber from linear to elliptical and circular and back to linear in a periodic behavior [1].

$$L_B=\frac{2\pi}{\Delta\beta} \tag{12}$$

Constant spin profile is given by;

$$\alpha(z)=\alpha_0 z=\frac{2\pi}{p}z \tag{13}$$

where α_0 is the constant spin rate and p is the spin period, i.e. the length of a full spin along the z axis.

When the constant spin function (13) is applied to the fiber, DGD is given by

$$\tau(z)=\frac{1}{\omega\sqrt{1+4L_B^2/p^2}}\times\sqrt{\left(\frac{2\pi}{L_B}z\right)^2+\frac{2L_B/p}{\sqrt{1+4L_B^2/p^2}}\sin\left(\sqrt{1+4L_B^2/p^2}\,\frac{2\pi}{L_B}z\right)} \tag{14}$$

The oscillating term can be neglected with respect to linear growth term. With this approximation (14) takes the form

$$\tau(z)\approx\frac{2\pi z}{\omega L_B\sqrt{1+4L_B^2/p^2}} \tag{15}$$

One can determine the effectiveness of PMD reduction of a spin function with PMD reduction factor *PMDRF*, which is the ratio between DGDs of spun and unspun fibers [8].

$$PMDRF = \frac{\tau}{\tau_\omega} \qquad (16)$$

DGD of unspun fiber is given by

$$\tau_\omega = \frac{2\pi z}{\omega L_B} \qquad (17)$$

With (15) and (17), *PRMDRF* of a constant spin profile has the following form

$$PMDRF_{cons} = \frac{1}{\sqrt{1+4L_B^2/p^2}} \qquad (18)$$

It can be noted that *PMDRF* of a constant spin depends on the ratio L_B/p. When the L_B is longer that p, constant spin profile is effective in reducing DGD. In Fig. 2, PMDRF is given with respect to p for four values of L_B [17].

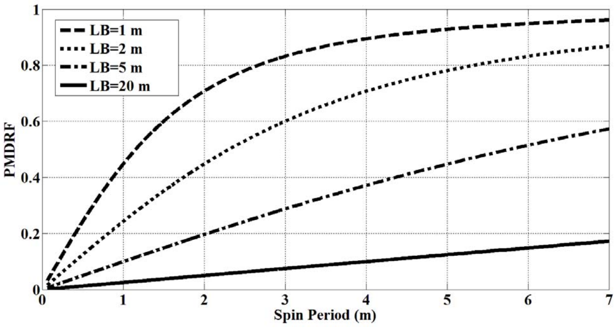

Fig. 2. Evolution of PMDRF as a function of spin period for different L_B values

3.2. Bidirectional Spinning

The second spin type is the bidirectional spin where fiber is spun clockwise and counterclockwise alternately with a periodical spin function. In the case of bidirectional spinning, one needs two parameters to define the spin profile, i.e. the spin amplitude α_0 and the spin period p. Now let's define several periodic spin profiles to analyze bidirectional spinning. Comparative analysis of their ability of reducing PMD is given in the following section.

$$\alpha(z) = \alpha_0 \cos(\eta z) \qquad (19)$$

Sinusoidal spin function takes the form of (19) where $\eta = 2\pi/p$ is the angular frequency of spin profile. One can calculate DGD using (11) for the sinusoidal spin profile as

$$\tau(z) = \tau_\omega \left| \int_0^z e^{-i\frac{2\alpha_0 \sin(\eta z')}{\eta}} dz' \right| \qquad (20)$$

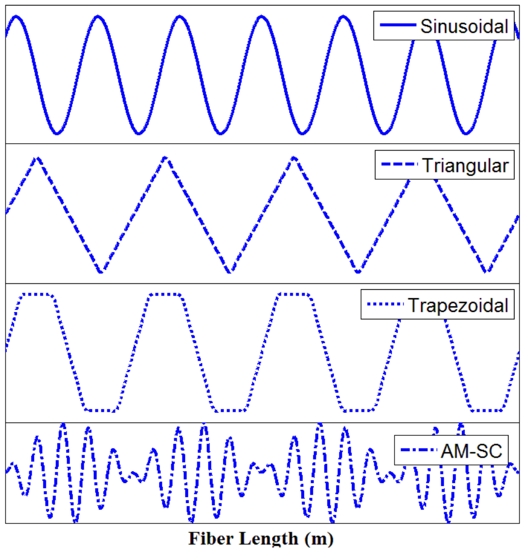

Fig. 3. Periodic spin profiles

The integration in (20) can be evaluated analytically [7] and PMDRF of sinusoidal spin profile takes the form of a Bessel function.

$$PMDRF_{\sin} = \left| J_0\left(\frac{2\alpha_0}{\eta}\right) \right| \qquad (21)$$

Using (21) one can describe PMDRF as a function of spin amplitude and spin period.

Triangular spin profile is defined by

$$f_{tri}(z) = \alpha_0 \begin{cases} -2 - \frac{2z}{\pi}, & -\pi < z' < \frac{-\pi}{2} \\ \frac{2z}{\pi}, & -\frac{\pi}{2} < z' < \frac{\pi}{2} \\ 2 - \frac{2z}{\pi}, & \frac{\pi}{2} < z' < \pi \end{cases} \qquad (22)$$

To use triangular spin profile in (11) one can expand (22) in Fourier series as

$$f_{tri}(z) = \frac{8}{\pi^2}(\sin(\eta z) - \frac{\sin(3\eta z)}{9} + \frac{\sin(5\eta z)}{25} - \frac{\sin(7\eta z)}{49} + \frac{\sin(9\eta z)}{81} - \frac{\sin(11\eta z)}{121} + ...) \qquad (23)$$

Trapezoidal spin profile takes the form given in (24) similar to the case of the triangular spin.

Expanding (24) in Fourier series, trapezoidal function can also be used in (11).

$$f_{trap} = \alpha_0 \begin{cases} -4 - \dfrac{4z}{\pi}, & -\pi \le z < \dfrac{-3\pi}{4} \\ -1, & \dfrac{-3\pi}{4} \le z < \dfrac{-\pi}{4} \\ \dfrac{4z}{\pi}, & \dfrac{-\pi}{4} \le z < \dfrac{\pi}{4} \\ +1, & \dfrac{\pi}{4} \le z < \dfrac{3\pi}{4} \\ 4 - \dfrac{4z}{\pi}, & \dfrac{3\pi}{4} \le z < \pi \end{cases} \quad (24)$$

With Fourier series representation of any periodic function, one can use (11) to obtain DGD of spun fibers.

$$f_{trap}(z) = \frac{8\sqrt{2}}{\pi^2}(\sin(\eta z) + \frac{\sin(3\eta z)}{9} - \frac{\sin(5\eta z)}{25} - \frac{\sin(7\eta z)}{49} + \frac{\sin(9\eta z)}{81} + \frac{\sin(11\eta z)}{121} - ...) \quad (25)$$

Modulated signals can be used as spin profiles like

$$\alpha(z) = \alpha_0 \sin(2\pi fz)\sin\left(\frac{2\pi z}{p}\right) \quad (26)$$

(26) represents AM-SC spin function where f is the carrier frequency. The AM-SC spin profiles have multiple Fourier frequency components. PMD reduction can be achieved over a wide range of L_B [8].

4. Simulation Model and Results

Based on analytical solutions described above DGD reducing ability of spin profiles has been simulated using MATLAB to indicate the relationship between spin parameters and the DGD reduction. We have comparatively analyzed PMD reduction ability of different spin profiles introduced above. The simulation conditions can be listed as *5 m* beat length L_B, *1 m* fixed spin period p, *1550 nm* operating wavelength λ, $1\ m^{-1}$ carrier frequency of AM-SC function where $f=1/p_{carrier}$ and $0.01\ m^{-1}$ message frequency $f_{message}$.

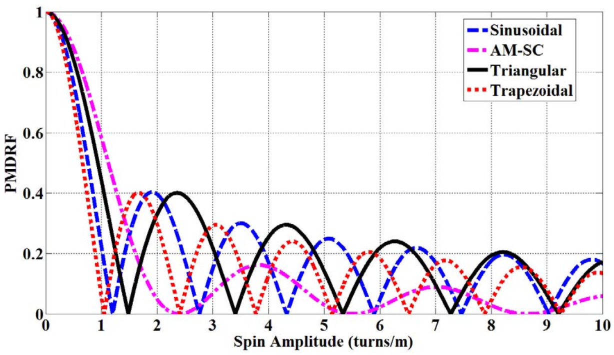

Fig. 4. PMDRF as a function of spin amplitude for four spin profiles, $p = 1\ m$

The PMDRF simulation has been carried out for values of spin amplitude from *0 turns/m* to *10 turns/m* with $1x10^{-3}$ *turns/m* intervals. And DGD simulations have been performed for short length fibers up to *10 m* with *1 mm* steps.

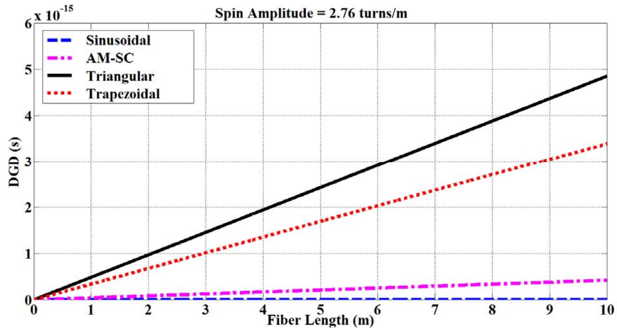

Fig. 5. DGD variation as a function of fiber length α_0*=2.76 turns/m, p=1 m,* L_B*=5 m*

Fig. 4 shows that for fixed spin period and increasing values of spin amplitude, PMDRF function has tendency to decrease with oscillations and to reduce PMD. One may choose spin amplitude as α_0*=2.76 turns/m* for sinusoidal spin function or as α_0*=3.40 turns/m* for triangular spin function.

For fixed spin amplitude of a_0*=2.76 turns/m*, Fig. 5 shows DGD variation as a function of fiber length and sinusoidal spin offers best PMD reduction performance among all spin profiles.

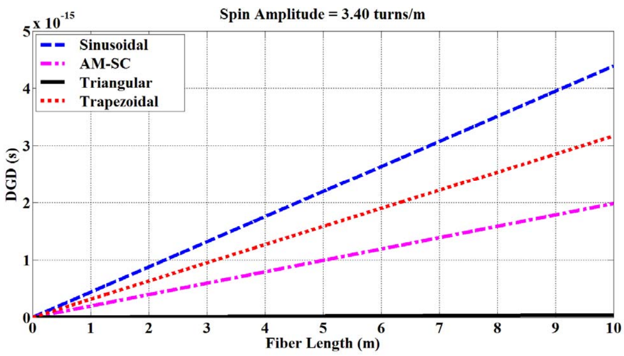

Fig. 6. DGD variation as a function of fiber length α_0*=3.40 turns/m, p=1 m,* L_B*=5 m*

When the value of spin amplitude is changed to *3.40 turns/m*, triangular spin profile gives better PMD reduction than other profiles as shown in Fig. 6.

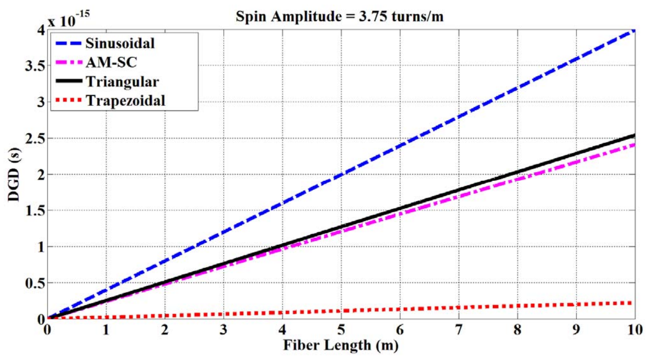

Fig. 7. DGD variation as a function of fiber length α_0*=3.75 turns/m, p=1 m,* L_B*=5 m*

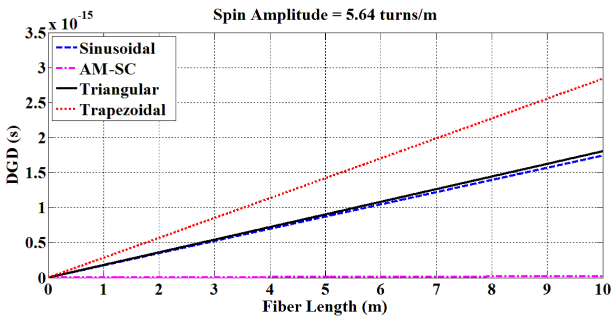

Fig. 8. DGD variation as a function of fiber length
α_0=*5.64 turns/m, p=1 m,* L_B=*5 m*

Fig. 7 and Fig. 8 compares DGD reduction of spin profiles for α_0=*3.75 turns/m* and α_0=*5.64 turns/m* respectively. In the first case trapezoidal function gives better PMD reduction and for latter case AM-SC spin profile has better results.

(21) shows that PMDRF is also proportional to the spin period. Thus PMDRF may change for different values of *p*.

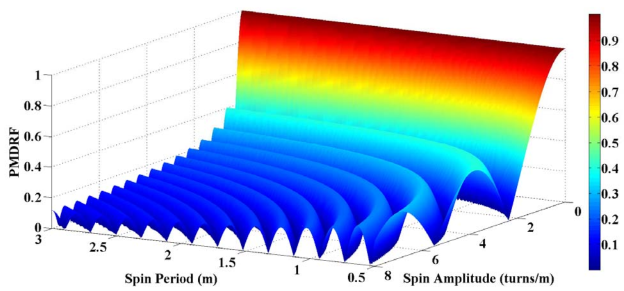

Fig. 9. PMDRF as a function of spin period and spin amplitude for sinusoidal spin profile

PMDRF function for sinusoidal spin profile with respect to spin parameters is shown in Fig. 9. We have found that when spin amplitude and spin period are increasing, PMDRF shows a decreasing characteristic with oscillations.

5. Conclusion

PMD properties of spun fibers are analyzed and PMD reducing abilities of different spin profiles are compared in this paper. The analytical solutions show that PMDRF changes with respect to spin parameters, i.e. the spin amplitude and the spin period. The results indicate that spin parameter values minimizing PMDRF function give best DGD reduction results.

We have found that α_0=*2.76 turns/m* and *p=1 m* are good spin parameter values for DGD reduction with sinusoidal spin profiles.

Results show that triangular spin function gives better DGD reduction with α_0=*3.40 turns/m* and *p=1 m* values. Also α_0=*3.75 turns/m* and *p=1 m* spin parameters are required for trapezoidal spin profile for low PMD fiber manufacturing while for AM-SC spin profile α_0=*5.64 turns/m* and *p=1 m* are essential spin parameters to reduce DGD.

Further studies are being focused on to optimize the spin process. It is expected that ultra-low-PMD fibers can be available meeting the PMD requirements for high-bit-rate optical fiber communication systems.

6. References

[1] G. P. Agrawal, "Fiber-optic comunication systems", 4th Edt. John Wiley & Sons Inc., USA, 2010.

[2] A. J. Barlow, J. J. Ramskov-Hansen, and D. N. Payne, "Birefrigence and polarization mode dispersion in spun single mode fibers", *Appl. Opt.*, vol. 20, no. 17, pp. 2962–2968, 1981.

[3] A. Galtarossa, Y. Jung, J. Kim, B. H. Lee, K. Oh, U. C. Paek, L. Palmieri, A. Pizzinat, L. Schenato, and C. G. Someda, "Low polarization mode dispersion measurements in ad hoc drawn spun fibers", *Opt. Fiber Technol.*, vol. 12, no. 4, pp. 323–327, 2006.

[4] L. Palmieri, T. Geisler, and A. Galtarossa, "Characterization of strongly spun fibers with spin rate exceeding OFDR spatial resolution", *2011 Opt. Fiber Commun. Conf. Expo. Natl. Fiber Opt. Eng. Conf.*, pp. 1–3, 2011.

[5] A. Galtarossa, P. Griggio, L. Palmieri, A. Pizzinat, and L. Schenato, "Polarization properties of randomly-birefringent spun fibers", *Opt. Fiber Technol.*, vol. 12, pp. 205–216, 2006.

[6] L. Palmieri, T. Geisler, and A. Galtarossa, "Effects of spin process on birefringence strength of single-mode fibers", *Opt. Express*, vol. 20, no. 1, p. 1, 2012.

[7] X. Chen, M. J. Li, and D. A. Nolan, "Polarization mode dispersion of spun fibers: an analytical solution", *Opt. Lett.*, vol. 27, no. 5, pp. 294–296, 2002.

[8] D. A. Nolan, X. Chen, and M. J. Li, "Fibers with low polarization mode dispersion", *J. Light. Technol.*, vol. 22, no. 4, pp. 1066–1077, 2004.

[9] M. J. Li and D. A. Nolan, "Fiber spin profile designs for producing fibers with low polarization mode dispersion", *Opt. Lett.*, vol. 23, no. 21, pp. 1659–1661, 1998.

[10] X. Chen, T. L. Hunt, M. J. Li, and D. A. Nolan, "Properties of polarization evolution in spun fibers", *Opt. Lett.*, vol. 28, no. 21, pp. 2028–2030, 2003.

[11] M. J. Li, X. Chen, and D. A. Nolan, "Ultra low PMD fibers by fiber spinning", *Opt. Fiber Commun. Conf. 2004*, vol. 2, pp. 3–5, 2004.

[12] J. Wen, W. Liu, Y. Huang, Y. Liu, and Y. Luo, "Spun-related effects on optical properties of spun silica optical fibers", *J. Light. Technol.*, vol. 33, no. 12, pp. 2674–2678, 2015.

[13] L. Palmieri and a. Galtarossa, "A preliminary analysis of spin in few-mode optical fibers", *39th Eur. Conf. Exhib. Opt. Commun. (ECOC 2013)*, no. 1, pp. 849–851, 2013.

[14] L. Palmieri, T. Geisler, and A. Galtarossa, "About the differential group delay of spooled spun fibers", *J. Light. Technol.*, vol. 26, no. 22, pp. 23–25, 2012.

[15] L. Palmieri, T. Geisler, and A. Galtarossa, "Experimental evidences of independence between birefringence modulus and spin rate in periodically spun fibers", *2011 37th Eur. Conf. Exhib. Opt. Commun.*, no. 5, pp. 1–3, 2011.

[16] T. M. Oktem, "A low complexity adaptive framework for the rate and performance enhancement of PMD limited fiber optic communication systems", MSc Thesis, Electrical & Computer Eng., Koç University, İstanbul, Turkey, 2006.

[17] C. R. Menyuk and A. Galtarossa, "Polarization mode dispersion", 1st ed. Springer, USA, 2005.