



## ON DIFFERENTIAL GEOMETRY OF THE LORENTZ SURFACES

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### Abstract

In this paper we have defined the sign functions  $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5$  and the vector fields  $X_u, X_v, n_u$  and  $n_v$  which have taken derivatives with  $(u,v)$  parameters of the tangent vector field  $X$  of any surface in Lorentz space and we get fundamental forms, Weingarten equations, Olin-Rodrigues and Gauss formulae. Beside these we calculate Gauss and mean curvatures.

*Keywords: Lorentz Surface, Fundamental Forms, Curvatures, Weingarten Formulae*

### Preliminaries

It is well known that in a Lorentzian Manifold we can find three types of submanifolds: Space-like (or Riemannian), time-like (Lorentzian) and light-like (degenerate or null), depending on the induced metric in the tangent vector space. Lorentz surfaces has been examined in numerous articles and books. In this article, however, we have examined some characteristics belonging to the surface by making some special choices on tangent space along the coordinate curves of the surface. Let  $\mathbb{R}^3$  be endowed with the pseudoscalar product of  $X$  and  $Y$  is defined by

$$\langle X, Y \rangle = x_1 y_1 + x_2 y_2 - x_3 y_3 \quad X = (x_1, x_2, x_3), Y = (y_1, y_2, y_3)$$

$(\mathbb{R}^3, \langle \cdot, \cdot \rangle)$  is called 3-dimensional Lorentzian space denoted by  $L^3$  [1]. The Lorentzian vector product is defined by

$$X \times Y = \begin{vmatrix} e_1 & e_2 & -e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

A vector fields  $X$  in  $L^3$  is called a space-like, light-like, time-like vector field if  $\langle X, X \rangle > 0$ ,  $\langle X, X \rangle = 0$   $\langle X, X \rangle < 0$  accordingly. For  $X \in L^3$ , the norm of  $X$  defined by

$$\|X\| = \sqrt{|\langle X, X \rangle|}$$

and  $X$  is called a unit vector if  $\|X\| = 1$  [2].