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GAUSS AND CODAZZI-MAINARDI FORMULAE

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Abstract

In this paper we have defined ε_i sign functions using the vector fields X_u, X_v, n_u and n_v which have taken derivatives with (u,v) parameters of tangent vector X of any surface in Lorentz space and we obtain Gauss and Codazzi-Mainardi Gauss formulae of the surface.

Preliminaries

It is well known that in a Lorentzian Manifold we can find three types of submanifolds: Space-like (or Riemannian), time-like (Lorentzian) and light-like (degenerate or null), depending on the induced metric in the tangent vector space. Lorentz surfaces has been examined in numerous articles and books. In this article, however, we have examined some characteristics belonging to the surface by making some special choices on tangent space along the coordinate curves of the surface. Let IR^3 be endowed with the pseudoscalar product of X and Y is defined by

$$\langle X,Y\rangle = x_1y_1 + x_2y_2 - x_3y_3 \qquad X=(x_1,x_2,x_3 \quad Y=(y_1,y_2,y_3)$$

$(IR^3,\langle,\rangle)$ is called 3-dimensional Lorentzian space denoted by L^3 1]. The Lorentzian vector product is defined by

$$X\times Y = \begin{vmatrix} e_1 & e_2 & -e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

A vector X in L^3 is called a space-like, light-like, time-like vector if $\langle X,X\rangle > 0$, $\langle X,X\rangle = 0$ or $\langle X,X\rangle < 0$ accordingly. For $X \in L^3$, the norm of X defined by

$$\|X\| = \sqrt{|\langle X,X\rangle|}$$

and X is called a unit vector if $\|X\| = 1$[2].

1. INTRODUCTION

Definition 1.1. A symmetric bilinear form b on vector space V is

i) positive [negative] definite provided $v \neq 0$ implies $b(v,v) > 0$ [< 0]

ii) positive [negative] semi-definite provided $v \geq 0$ [$v \leq 0$] for all $v \in V$

iii) non-degenerate provided $b(v,w) = 0$ for all $w \in V$ implies $v = 0$ [1].