



## A GENERALIZED DE SITTER LIKE NONSINGULAR COSMOLOGICAL SOLUTION FOR THE INFLATIONARY ERA AND THE FRIEDMANN ERA IN THE VERY EARLY UNIVERSE

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### ABSTRACT

A generalized De Sitter like nonsingular cosmological solution for the inflationary era and the Friedmann era in the very early universe is found to the ten dimensional effective Einstein field equations with the effective perfect fluid source. The validity criteria of this solution in the inflationary era does not satisfy the null energy condition (NEC). This solution contains the quantum correction function which explains the phase transition of the universe from the inflationary phase to the Friedmann phase in nowadays. The actual universe space and the internal space are the Ricci flat spaces.

**Key Words:** *Nonsingular cosmology, Ricci flat spaces, NEC*

## ERKEN EVRENDE ŞİŞMELİ ÇAĞ VE FRIEDMANN ÇAĞI İÇİN TEKİL OLMAYAN DE SITTER TİPİ GENELLEŞTİRİLMİŞ BİR KOZMOLOJİK ÇÖZÜM

### ÖZET

Erken evrende şişmeli çağ ve Friedmann çağı için on boyutlu etkin mükemmel akışkan kaynaklı etkin Einstein alan denklemlerine de Sitter tipi tekil olmayan genelleştirilmiş bir kozmolojik çözüm bulunur. Bu çözümün geçerlilik ölçütleri şişmeli çağda sıfır enerji şartını (NEC) sağlamaz. Bu çözüm, evrenin şişmeli çağdan günümüzdeki Friedmann fazına faz geçişini açıklayan bir kuantum düzeltme fonksiyonunu içerir. Gerçek evren uzayı ve iç uzay Ricci düz uzaylardır.

**Anahtar Kelimeler:** *Tekil olmayan kozmoloji, Ricci düz uzaylar, NEC*

### 1.INTRODUCTION

First of all, it is considered that the very early universe, at least the inflationary era is described by the ten dimensional cosmological metric and the effective Einstein equations [1], [4], [6]. It is assumed that in the considered era of the universe, at the zeroth order of the effective string theory, the total effect of the scalar field (the dilaton), the axion fields (Kalb-Ramond fields) and the others (included the total effects of quantum corrections) can be effectively maintained by taking an effective perfect fluid energy-stress tensor as the source of the effective Einstein field equations. It is known that these fields break the null energy condition (NEC) under some certain constraints.

The various cosmological solutions of this kind have previously been given in the literature [3],[6],[5]. In addition to the vast literature on this subject, the detailed global and the local structure of the nonsingular universe has been explained in a recent work [7]. However, it is desired to find an acceptable special nonsingular solution which is also important for the quantum gravity at the same time. For these reasons, this paper is presented. This work generalizes the previous work [8].

The components of the four velocity vector  $u^a$  in the energy-stress tensor were selected in the form of (1,0,0,0). For these reason, the consequences of this paper are valid in any physical frame, namely, the results are valid in general.

## 2. THE FIELD EQUATIONS, THE COSMOLOGICAL SOLUTIONS

For the three dimensional Ricci flat actual universe and the six dimensional compact Ricci flat internal space, the line element and the Einstein equations are given by

$$ds^2 = -dt^2 + R(t)^2 (dx^2 + dy^2 + dz^2) + S(t)^2 (dy_1^2 + dy_2^2 + dy_3^2 + dy_4^2 + dy_5^2 + dy_6^2) \quad (1)$$

$$3 (1/R)^2(dR/dt)^2 + 15 (1/S)^2(dS/dt)^2 + 18 (1/R)(dR/dt) (1/S)(dS/dt) = \kappa (\rho - \lambda) \quad (2)$$

$$(1/R)^2(dR/dt)^2 + 15 (1/S)^2(dS/dt)^2 + 12 (1/R)(dR/dt) (1/S)(dS/dt) + 2 B + 6 C = -\kappa(\lambda+p) \quad (3)$$

$$3 (1/R)^2(dR/dt)^2 + 10 (1/S)^2(dS/dt)^2 + 15 (1/R)(dR/dt) (1/S)(dS/dt) + 3 B + 5 C = -\kappa(\lambda+r) \quad (4)$$

where  $\kappa$  denotes the effective coupling constant,  $R(t)$  is the actual space scale factor,  $S(t)$  is the internal space scale factor,  $t$  is the cosmic time (in the units in which the velocity of light equals to unit),  $(x,y,z)$  are the actual space coordinates,  $(y_1, y_2, y_3, y_4, y_5, y_6)$  are the internal space coordinates and the following abbreviations has been used

$$B = (1/R)(d^2R/dt^2)$$

$$C = (1/S)(d^2S/dt^2).$$

Furthermore, the above effective Einstein field equations also satisfy the following covariant conservation relation

$$(d\rho/dt) + 3 (1/R)(dR/dt)(\rho+p) + 6 (1/S)(dS/dt) (\rho+r)=0, \quad (5)$$

where and in everywhere  $\rho$  denotes the energy density of the universe  $p$  denotes the actual universe pressure,  $r$  denotes the internal universe pressure and  $\lambda$  denotes the positive cosmological constant [5].

The cosmological solutions are considered in the form of

$$R=(\Phi)^{p1} \quad \text{and} \quad S=(\Phi)^{q1}$$

where  $\Phi$  is the function of the cosmic time. For the solution, the  $\Phi, p1, q1$  satisfy the following equations

$$p1 = 5/9, q1 = -1/9, 3 p1 + 6 q1 = 1, 3 p1^2 + 6 q1^2 = 1, \quad (6)$$

$$(4/9) (1/\Phi)(d^2 \Phi /dt^2) = -\kappa(\lambda+p), \quad (7)$$

$$(10/9) (1/\Phi)(d^2 \Phi /dt^2) = -\kappa(\lambda+r). \quad (8)$$

As the solution,  $\lambda = \rho$  has been taken as in [2]. This means that the universe is in the Bose-Einstein condensed state with the zero energy. Furthermore, in this solution the following expressions are valid :

$$\rho = \rho_0 / 3, p = -(3/5) p_0 + f, r = 3 r_0 + (5/2) f, \rho_0 - 3 p_0 - 6 r_0 = 0, \quad (9)$$

where  $\rho_0, p_0$  ve  $r_0$  are the universal cosmic constants and  $f$  comes from the quantum corrections to the Einstein's field equations and  $f$  determines the phase transition of the universe from the inflationary phase to the Friedmann phase in nowadays.

$f$  has the form of  $f = ((3/5)p_0 - \rho_0/3) H(t - t_0)$ , where  $H(t - t_0)$  is the Heaviside unit step function which equals to zero for  $t < t_0$  and equals to unit for  $t > t_0$ . The  $t_0$  denotes the spontaneous phase transition moment. For  $t < t_0$  the universe is in the inflationary era and for  $t > t_0$  the universe is in the Friedmann era.

### 3. CONCLUDING REMARKS

In this paper, a generalized nonsingular De Sitter like inflationary cosmological solution has been given to the effective Einstein field equations. It can be accepted as an appropriate cosmological solution for the Bose-Einstein condensed universe. This solution is also important for the effective string theory and quantum gravity because of this solution is related with the vacuum solutions of the Einstein's field equations so that the solution parameters satisfy the constraints of  $p_1 = 5/9$ ,  $q_1 = -1/9$ ,  $3p_1 + 6q_1 = 1$ ,  $3p_1^2 + 6q_1^2 = 1$  and this solution explains the phase transition of the universe from the inflationary phase to the Friedmann phase in nowadays

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